Simulation of butterfly flapping with the method of dipole domains

Galina Ya. Dynnikova*, Sergey V. Guvernyuk*, Dmitry A. Syrovatskiy*

* Lomonosov Moscow State University
Institue of Mechanics,
Michurinsky pr. 1, 119192, Moscow, Russia
e-mail: dyn@imec.msu.ru, guv@imec.msu.ru, talismanium@gmail.com

ABSTRACT

A numerical mesh-free method of dipole domains [1,2] is used for simulation of a butterfly flapping model. This method is based on the representation of a vortex field by the set of dipole particles. The vector function D describes density of dipole moments in accordanse with Navier-Stokes or Euler equations [3]. The butterfly model consists of two flat plates with a common edge performing harmonic oscillations in two planes. New mechanism of the thrust performing is proposed.

Key words: Mesh-free numerical method, three dimensional flow, dipole particles, butterfly model, mechanism of the thrust performance.

1. INTRODUCTION

The mesh-free particles-based methods are effective for modeling the flows with intensively changing boundaries. In the grid based methods two strategies are usually utilized: morphing grids and overset grids. The first approach is not applicable when the computational domain variation is large. The overset grid technology suffers from the low accuracy of computations which is caused by non conservative character of the interpolation between grids. Mesh-free methods could be a good alternative to grid based techniques for such problems.

Among the mesh-free methods, the vortex methods have an advantage in modeling incompressible flows in an unbounded space, since the region with essentially non-zero vorticity has a small volume. In addition, the boundary conditions at infinity are automatically provided.

Simulation of 3D vortex flow in 3-D space has the problem of the representation of three-dimensional vortex field by discrete elements. This field must be divergence-free as a curl of velocity field. But when the discrete vortex particles are used, this property can be destroyed. The velocity field which the vortex particle induces in accordance with Biot-Savart formula has non-zero vorticity in the whole space but not only in the localization of the particle. If the set of the vortex particles does not form a divergence-free vector field then the rotor of the induced velocity field does not coincide with this vector field. This leads to errors in the calculation if special measures are not taken. Therefore hybrid methods are often applied with combination of the Eulerian and Lagrangian approaches [4]. After the particles have been moved, their intensities are recalculated at Euler mesh for recovering the solenoidality at each step. This procedure enforces to build grids, and can increase the numerical viscosity.

In this work the dipole particles are used for simulating of the 3-D vortex field. This representation provides a solenoidality of the vortex field. The fully lagrangian method of Dipole Domains (DD) is developed in [1]. Dipole distributions are widely used in hydrodynamics to calculate the potential flows (double-layer potential). The idea to construct a numerical method based on the dipole particles was suggested by Yanenko, Veretentsev and Grigoriev [5]. However, numerical implementation hasn't been performed. Chefranov [6] used the point dipoles to model the vorticity in an ideal fluid for analyzing the mechanisms of turbulence and turbulent viscosity. It has been shown that interaction of the point dipoles in an ideal fluid can lead to explosive growth of localized vorticity. The vortex dipoles were applied in papers [7-9] for the simulation of the inviscid vortex flow and analyzing of the turbulence. In the method of Dipole Domains the smooth dipole particles are used. Viscous interaction of the particles can be taken into account.

2. GOVERNING EQUATIONS

The method of dipole domains is based on the equation for the vector function **D** called as density of dipole moments

$$\frac{\partial \mathbf{D}}{\partial t} = \mathbf{V}\times\mathbf{\Omega} + \nu\nabla^2\mathbf{V} - \nabla\left(\mathbf{V}\mathbf{D} - \nu\nabla\mathbf{D}\right), \tag{1}$$

where **V** is fluid velocity, $\mathbf{\Omega} = \nabla\times\mathbf{V}$, $\nabla\cdot\mathbf{V} = 0,$, ν is kinematic viscosity. Applying curl to equation (1) one can see that evolution of the field $\nabla\times\mathbf{D}$ obeys the same equation as the vorticity $\mathbf{\Omega}$. Hence at the equivalent boundary conditions $\nabla\times\mathbf{D} = \mathbf{\Omega}$. The velocity field **V** is a divergence-free part of **D**. It can be expressed via **D** with the help of Biot-Savarat formula. In infinite space τ it has the form

$$\mathbf{V}(\mathbf{R}) = \frac{1}{4\pi}\int \frac{\mathbf{r}-\mathbf{R}}{|\mathbf{r}-\mathbf{R}|^3}\times\left(\nabla\times\mathbf{D}\right)\mathrm{d}\tau + \mathbf{V}_\infty, \quad \mathbf{r}\in\tau. \tag{4}$$

Expression (4) can be transformed to following

$$\mathbf{V}(\mathbf{R}) = \frac{2}{3}\mathbf{D}(\mathbf{R}) + \frac{1}{4\pi}\int\left(-\frac{\mathbf{D}}{|\mathbf{r}-\mathbf{R}|^3} + \frac{3(\mathbf{r}-\mathbf{R})\left((\mathbf{r}-\mathbf{R})\cdot\mathbf{D}\right)}{|\mathbf{r}-\mathbf{R}|^5}\right)\mathrm{d}\tau. \tag{5}$$

Here the integral is taken in the principal value. The integrand is the velocity which the point dipole located in **r** induces in the point **R**. That is why we call function **D** as dipole density.

The use of the field **D** has an advantage over the natural variables because the numerical scheme can be constructed in such a way that **D** will have a non-zero value mainly in the wakes behind the bodies. The advantage over the vortex methods is that the solenoidality of the vorticity field is provided automatically.

For the hydrodynamic force calculation we use expression of the hydrodynamic impulse of the flow via **D**. In accordance with [10] in the case of the infinite space and finite distribution of the vorticity, the hydrodynamic impulse I is equal to

$$\mathbf{I} = \frac{1}{2}\int \mathbf{r}\times\mathbf{\Omega}\,\mathrm{d}\tau$$

Taking into account $\nabla\times\mathbf{D} = \mathbf{\Omega}$ and applying the general Stokes theorem we obtain

$$\begin{aligned}\mathbf{I} &= \frac{1}{2}\int \mathbf{r}\times\left(\nabla\times\mathbf{D}\right)\mathrm{d}\tau = \frac{1}{2}\oint_{S_\infty} \mathbf{r}\times\left(\mathbf{n}\times\mathbf{D}\right)\mathrm{d}S - \frac{1}{2}\int\left(\mathbf{D}\times\nabla\right)\times\mathbf{r}\,\mathrm{d}\tau = \\ &= -\frac{1}{2}\int\left(\mathbf{D}\cdot\nabla\right)\mathbf{r}\,\mathrm{d}\tau + \frac{1}{2}\int\mathbf{D}\left(\nabla\cdot\mathbf{r}\right)\mathrm{d}\tau = \int\mathbf{D}\,\mathrm{d}\tau\end{aligned} \tag{6}$$

Formulas (5), (6) are also valid in the case of flows around infinitely thin bodies, if they are represented as the distribution of **D**. The hydrodynamic force acting on the body is equal to

$$\mathbf{F}_H = -\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}t}$$

3. NUMERICAL METHOD

In this work we use the simplified model of the ideal incompressible flow around infinitely thin bodies. The field **D** is represented by the set of dipole particles with dipole moments $\boldsymbol{\zeta}_i$. Each particle is smoothed by function

$$\mathbf{D}_i(\mathbf{r}) = \begin{cases}\dfrac{\varsigma_i}{\varepsilon_i^3} f(\xi_i), & \xi < 1,\\ 0, & \xi \geq 1,\end{cases} \qquad \xi = \frac{|\mathbf{r}-\mathbf{r}_i|}{\varepsilon_i}, \quad f(\xi_i) = \frac{105}{16\pi}(1+3\xi)(1-\xi)^3. \tag{7}$$

Equation (1) is transformed to the following form

$$\frac{\partial \mathbf{D}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{D} = -(\mathbf{D}\cdot\nabla)\mathbf{V} - \mathbf{D}\times\mathbf{\Omega}$$

This equation corresponds to the particle movement at the velocity **V** with changing dipole moment

$$\frac{\partial \varsigma_i}{\partial t} = -(\varsigma_i\nabla)\mathbf{V} - \varsigma_i\times\mathbf{\Omega}\,.$$

The discrete formulas for the velocity are:

$$\mathbf{V}(\mathbf{R}) = \sum_n\left(-\frac{\varsigma_n a}{r'^3} + \frac{3b(\varsigma_n\mathbf{r}')\mathbf{r}'}{r'^5}\right) + \mathbf{V}_\infty,$$

$$\mathbf{r}' = \mathbf{R} - \mathbf{r}_n,\quad a = \eta_n - \sigma_n,\quad b = \eta_n - \frac{\sigma_n}{3},$$

$$\eta_n = \int_0^{r'/\varepsilon}\xi^2 f(\xi)\,d\xi,\quad \sigma_n = \frac{r'^3}{\varepsilon^3} f\left(\frac{r'}{\varepsilon}\right).$$

The rate of the dipole moment changing has the following discrete form

$$\frac{\mathrm{d}\varsigma_i}{\mathrm{d}t} = \sum_n \boldsymbol{\delta}_{in},$$

$$\boldsymbol{\delta}_{in} = \left(\frac{(\varsigma_i\varsigma_n)(3a - a'r')}{r'^5}\mathbf{r}' + \frac{3b\left((\varsigma_i\mathbf{r}')\varsigma_n + (\varsigma_n\mathbf{r}')\varsigma_i\right)}{r'^5} - \right.$$

$$\left. - \frac{3(5b - b'r')(\varsigma_n\mathbf{r}')(\varsigma_i\mathbf{r}')}{r'^7}\mathbf{r}'\right).$$

One can see that $\boldsymbol{\delta}_{in} = -\boldsymbol{\delta}_{ni}$, that is hydrodynamic impulse $\mathbf{I} = \sum_i \varsigma_i$ conserves if the bodies and external forces are absent in the flow.

The infinitely thin plates are simulated by the attached dipole particles. They have to provide the boundary non-flow condition. New free dipole particles shed into the wake from trailing edge satisfying the Kutta-Zhukovcky condition.

4. RESULTS AND DISCUSSION

The method is applied for simulation of the ideal incompressible flows around the model of butterfly wings. The particles are generated at the plates and detach from the trailing edges.

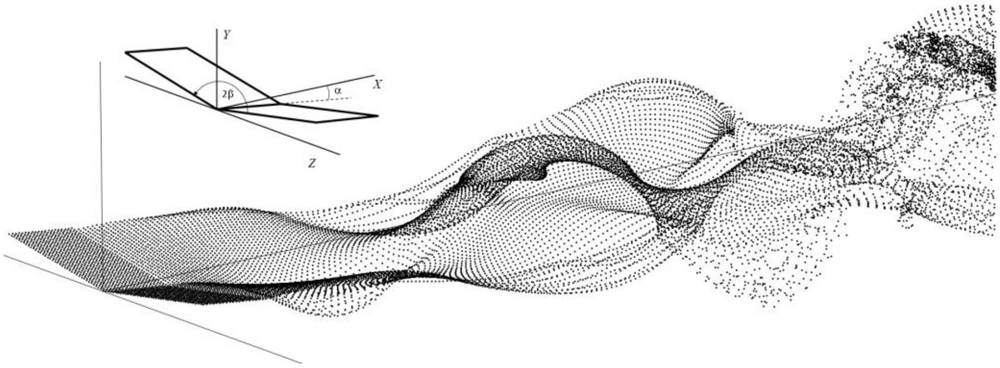

Figure.1 The wake behind the flapping wings of the butterfly model

The model of the butterfly wings consists of two plates with the common edge (see Fig.1). The plates perform angular oscillations about this edge according to law $\beta = \pi/2 - \gamma(t)$, where $\gamma = \gamma_0 \sin(2\pi f t)$. In addition, the entire system performs angular oscillations about the Z axis $\alpha = \alpha_1 + \alpha_0 \sin(2\pi f t + \varphi)$. A vortex wake behind the wings is shown In fig. 2 at $\alpha_0 = 10°$, $\alpha_1 = -5°$, °$\gamma_0 = 10°$, $\varphi = -150°$, Strouhal number Sh = 0.15. The time dependencies of the coefficients Cx, Cy, and the angles α and β are presented in fig.2. It can be seen that the average lifting force in this mode is positive, and the average resistance is negative, i.e. there is a propulsive force towards the flow.

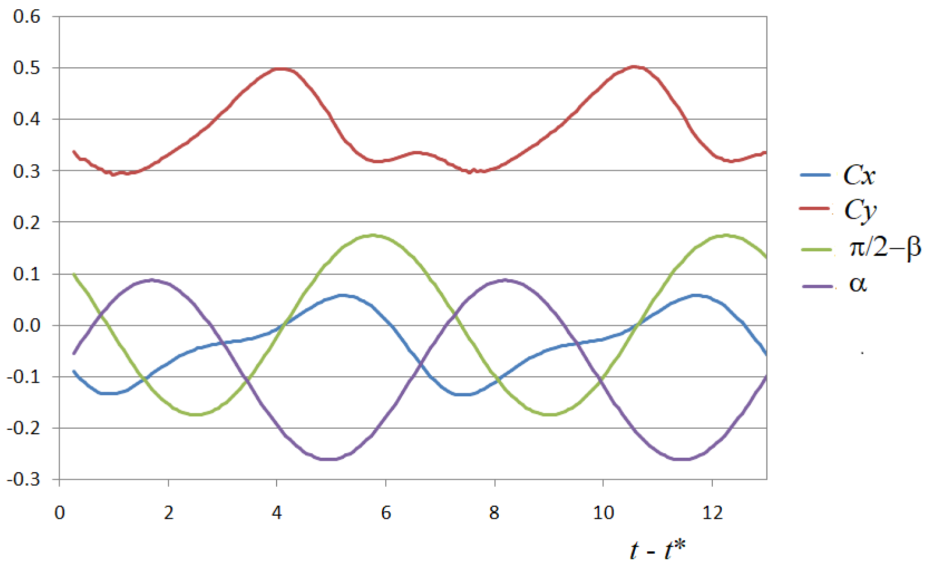

Figure 2. Time dependencies of the coefficients Cx, Cy, and the angles α and β

Most of investigators studying flapping wings find the reason of thrust performance in the vortex structures arising near the bodies such as reversive vortex street and vortex ring [11-16]. But it is more credible that these structures are not a cause but a consequence of thrust. We think that the reason of thrust is accelerated wing movement at an appropriate angle of attack. Let us consider the combined oscillations of a two-dimensional plate consisting of translational oscillation along Y-axis (see fig. 3) $y = y_0 \sin\left(\frac{2\pi}{T}t\right)$ and angular oscillation $\alpha = \alpha_0 \sin\left(\frac{2\pi}{T}t\right)$ around the leading edge (the left point).

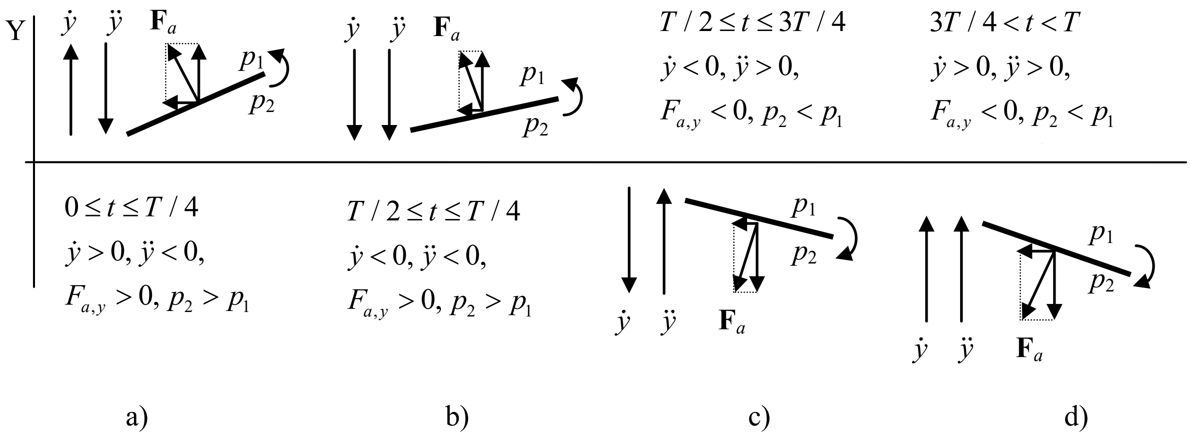

Figure 3. Scheme of the vectors directions in different oscillation phase

In the first quarter of the period, the plate moves up, but the acceleration is directed downwards (fig. 3a). When the body moves with acceleration, a force $\mathbf{F}_a$ arises associated with the added mass of fluid. This force is directed against acceleration, i.e. in this case, upwards. If the acceleration is large enough, this force dominates. This means that the pressure on the lower side of the plate is higher than on the top. The horizontal component of $\mathbf{F}_a$ is directed from the right to the left. The counter clock-wise vortex sheds from the trailing edge. In the second quarter of the period, the acceleration is also directed downward. The horizontal component of the force $\mathbf{F}_a$ is also directed from the right to the left. In the second half of the period, the acceleration changes sign. The angle of the plate changes the sign also. As a result, the horizontal component of the force conserves direction. Thus the interaction of the plate with the added mass of fluid creates a propulsion force. The pressure difference between the lower and upper sides of the plate determines the sign of the vortex shedding from the trailing edge. In the first half of the period this is the counter-clock-wise vortices, and clock-wise in the second half. In such a way a reversive vortex street arises.

4. CONCLUSIONS

The mesh-free dipole particles-based methods is developed and applied for simulation of the insect flapping wings. It is shown that the main cause of the thrust performance is interaction of wings with an added mass of fluid at the wings acceleration.

ACKNOWLEGMENT

The work was supported by the Russian Science Foundation (grant No. 18-71-00133).

REFERENCES

[1] Dynnikova G.Ya..*Calculation of three-dimensional flows of an incompressible fluid based on a dipole representation of vorticity*, Doklady Physics. Vol. 56, no. 3, p. 163–166 (2011).

[2] Syrovatskiy D., Dynnikova G., Guvernyuk S., Arutunyan G. *Using the dipole particles for simulation of 3d vortex flow of a viscous incompressible fluid*, IV International Conference on Particle-based Methods – Fundamentals and Applications PARTICLES 2015 Barcelona, Spain, 2015. p. 692–701.Cottet, G.H. and Koumoutsakos, P.D. *Vortex Methods: Theory and Practice.* Cambridge University Press, (2000).

[3] Oseledets, V.I., *On a new way of writing the Navier-Stokes equation. The Hamiltonian formalism*, Russ.Math.Surveys, 44, 210-211 (1989).

[4] Cottet, G.H. and Koumoutsakos, P.D. *Vortex Methods: Theory and Practice.* Cambridge University Press, (2000).

[5] Yanenko, N.N., Veretentsev, A.N. and Grigoryev, Yu.N., *In: Numerical methods of Mechanics of continuous medium*. Novosibirsk, Vol. 10, p. 144, (1979), in Russian.

[6] Chefranov, S.G., *The dynamics of point vortex dipoles and spontaneous singularities in three-dimensional turbulent flows.* Journal of Exp. And Theor. Phys. (ISSN 0044-4510), Vol. 93, pp. 151-158, (1987), in Russian.

[7] Buttke, T.F. and Chorin, A.J., *Turbulence calculations in magnetization variables*, Appl. Numer. Math., 12 (1993), pp. 47–54.

[8] Cortez, R., *On the accuracy of impulse methods for fluid flow*. SIAM J. Sci. Comput. Vol. 19, No. 4, pp. 1290–1302, July (1998).

[9] Newton, P.K., *The dipole dynamical system.*Discret and continuous dynamical systems. Supplement Volume (2005) pp. 692-699.

[10] Saffman, P. G., *Vortex Dynamics,* Cambridge University Press, (1992).

[11] Gong Ch., Han J.,.Yuan Z et all. *Numerical investigation of the effects of different parameters on the thrust performance of three dimensional flapping wings*. Aerospace and Technology. № 84, p. 431-445, (2019).

[12] Cros A., Llamas B. F., Hernández E. S., *Vortical patterns generated by flapping foils of variable*

ratio chord-to-thickness. Experiments in Fluids (2018) 59:152 https://doi.org/10.1007/s00348-018-2604-2

[13] Bomphrey R., Godoy-Diana R. *Insect and insect-inspired aerodynamics: unsteadiness, structural mechanics and flight control*. Elsevier, 2018, 30 (Décembre 2018), pp.26-32. <10.1016/j.cois.2018.08.003>. <hal-01972623>

[14] Godoy-Diana R., Aider J. L., Wesfreid J. E. *Transitions in the wake of a flapping foil*. Phys. Rev. E 77, 016308, (2008).

[15] Wang Sh., He G., and Liu T. *Estimating lift from wake velocity data in flapping flight*. J. Fluid Mech. (2019), vol. 868, pp. 501-537. doi:10.1017/jfm.2019.181

[16] Fuchiwaki, M., Kuroki, T., Tanaka, K. and Tabata, T., *Dynamic Behavior of the Vortex Ring Formed on a Butterfly Wing*, Exp. Fluids, (2013), 54:1450, 2013.